

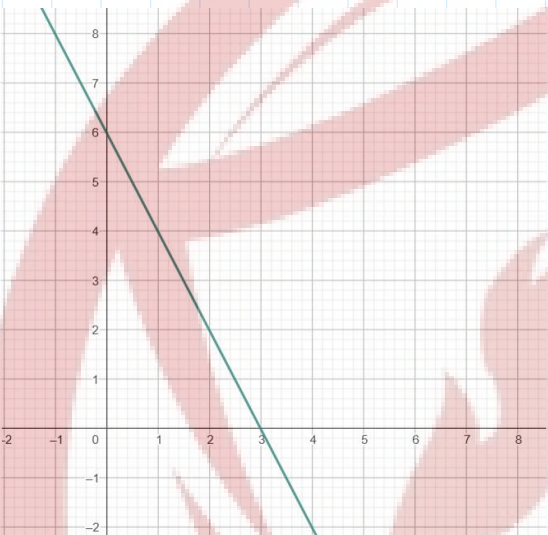
PRACTICA 3 CALCULO I ING VILDOZO

1) Indicar si los conjuntos de Pares ordenados forman una función (F. o no F.)

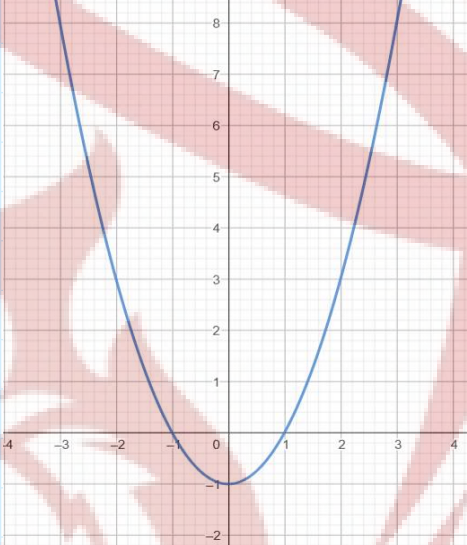
- a) $\{(3,0), (5,0), (7,0)\}$ Si es Función
- b) $\{(2,3), (3,4), (5,6)\}$ Si es Función
- c) $\{(1,2), (1,3), (1,4)\}$ No es Función
- d) $\{(0,0), (2,2), (4,4)\}$ Si es Función

2) Graficar las siguientes Relaciones y funciones en el plano coordenado.

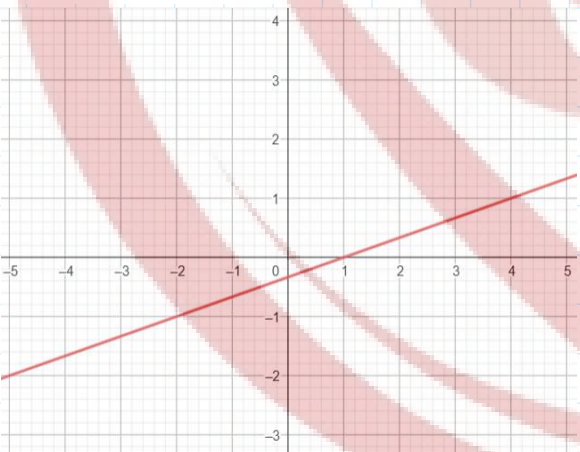
a) $y = 6 - 2x$



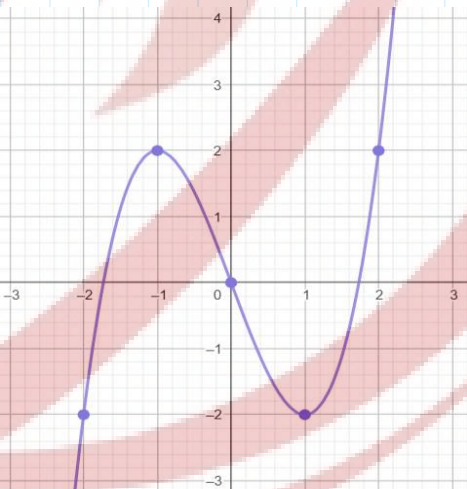
b) $(x^2 - 1) = y$



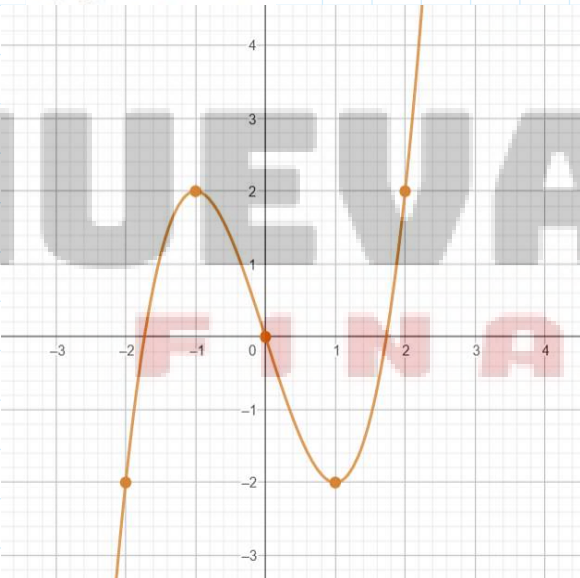
c) $y = \frac{(x-1)}{3}$



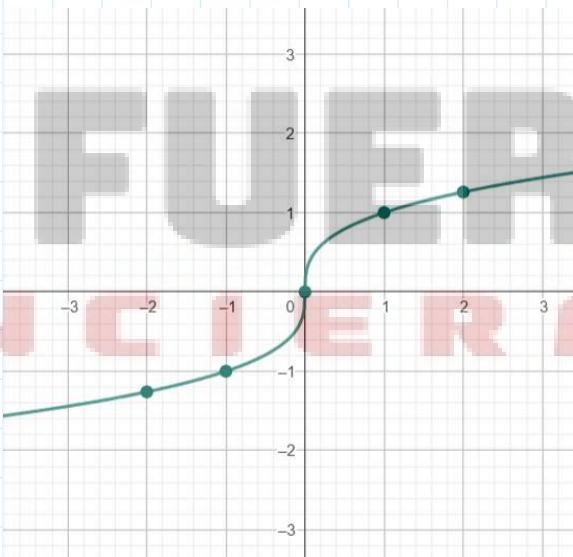
d) $x^3 - 3x - y = 0$



e) $y = x^3 - 3x$



f) $y = \sqrt[3]{x}$



3) Hallar Dominios (D_f) y Codominios (C_f)

a) $y = 3x + 6$

Dominio

$$y = 3x + 6$$

Codominio

$$\begin{aligned} y &= 3x + 6 \\ y - 6 &= 3x \\ x &= \frac{y - 6}{3} \end{aligned}$$

$$D_f : \forall x \in \mathbb{R}$$

$$C_f : \forall y \in \mathbb{R}$$

b) $y = \frac{5x}{x-3}$

Dominio

$$y = \frac{5x}{x-3}$$

$$\begin{aligned} x - 3 &\neq 0 \\ x &\neq 3 \end{aligned}$$

Codominio

$$\begin{aligned} y &= \frac{5x}{x-3} \\ y(x-3) &= 5x \\ xy - 3y &= 5x \\ xy - 5x &= 3y \\ x(y-5) &= 3y \\ x &= \frac{3y}{y-5} \end{aligned}$$

$$\begin{aligned} y - 5 &\neq 0 \\ y &\neq 5 \end{aligned}$$

$$D_f : \forall x \in \mathbb{R} / x \neq 3$$

$$C_f : \forall y \in \mathbb{R} / y \neq 5$$

c) $y = \frac{4x}{x-4}$

Dominio

$$y = \frac{4x}{x-4}$$

$$\begin{aligned} x - 4 &\neq 0 \\ x &\neq 4 \end{aligned}$$

Codominio

$$\begin{aligned} y &= \frac{4x}{x-4} \\ y(x-4) &= 4x \\ xy - 4y &= 4x \\ xy - 4x &= 4y \\ x(y-4) &= 4y \\ x &= \frac{4y}{y-4} \end{aligned}$$

$$\begin{aligned} y - 4 &\neq 0 \\ y &\neq 4 \end{aligned}$$

$$D_f : \forall x \in \mathbb{R} / x \neq 4$$

$$C_f : \forall y \in \mathbb{R} / y \neq 4$$

d) $y = \frac{1}{x-7}$

Dominio

$$y = \frac{1}{x-7}$$

$$\begin{aligned} x - 7 &\neq 0 \\ x &\neq 7 \end{aligned}$$

Codominio

$$\begin{aligned} y &= \frac{1}{x-7} \\ y(x-7) &= 1 \\ xy - 7y &= 1 \\ xy &= 1 + 7y \\ x &= \frac{1+7y}{y} \end{aligned}$$

$$y \neq 0$$

$$D_f : \forall x \in \mathbb{R} / x \neq 7$$

$$C_f : \forall y \in \mathbb{R} / y \neq 0$$

$$e) y = \frac{1}{x^2 - 6x + 5}$$

Dominio

$$y = \frac{1}{x^2 - 6x + 5}$$

$$\begin{aligned} x^2 - 6x + 5 &\neq 0 \\ (x-1)(x-5) &\neq 0 \\ x-1 &\neq 0; x-5 \neq 0 \\ x &\neq 1; x \neq 5 \end{aligned}$$

Codominio

$$y = \frac{1}{x^2 - 6x + 5}$$

$$\begin{aligned} y(x^2 - 6x + 5) &= 1 \\ x^2y - 6xy + 5y &= 1 \\ x^2y - 6xy + 5y - 1 &= 0 \end{aligned}$$

$$x = \frac{-(-6y) \pm \sqrt{(-6y)^2 - 4y(5y-1)}}{2y}$$

$$x = \frac{6y \pm \sqrt{36y^2 - 20y^2 + 4y}}{2y}$$

$$x = \frac{6y \pm \sqrt{16y^2 + 4y}}{2y}$$

Numerador

$$16y^2 + 4y \geq 0 \quad // (\frac{1}{4})$$

$$4y^2 + y \geq 0$$

$$y(4y+1) \geq 0 \Rightarrow \begin{aligned} &y \geq 0 \wedge 4y+1 \geq 0; \quad y \leq 0 \wedge 4y+1 \leq 0 \\ &y \geq 0 \wedge y \geq -\frac{1}{4}; \quad y \leq 0 \wedge y \leq -\frac{1}{4} \end{aligned}$$

$$y \geq 0; \quad y \leq -\frac{1}{4}$$

Denominador

$$2y \neq 0$$

$$y \neq 0$$

$$f) y = \frac{1}{\sqrt{x-5}}$$

Dominio

$$y = \frac{1}{\sqrt{x-5}}$$

$$\begin{aligned} x-5 &> 0 \\ x &> 5 \end{aligned}$$

Codominio

$$y = \frac{1}{\sqrt{x-5}}$$

$$\begin{aligned} y(\sqrt{x-5}) &= 1 \\ (\sqrt{x-5})^2 &= \left(\frac{1}{y}\right)^2 \end{aligned}$$

$$x-5 = \frac{1}{y^2}$$

$$x = \frac{1}{y^2} + 5$$

$$x = \frac{1-5y^2}{y^2}$$

$$y^2 = 0$$

$$y \neq 0$$

$$D_f: \forall x \in \mathbb{R} / x > 5$$

$$C_f: \forall y \in \mathbb{R} / y \neq 0$$

$$g) y = +\sqrt{x^2 - 6x + 8}$$

Domínio

$$y = \sqrt{x^2 - 6x + 8}$$

$$x^2 - 6x + 8 \geq 0$$

$$(x-2)(x-4) \geq 0$$

$$x-2 \geq 0 \wedge x-4 \geq 0; x-2 \leq 0 \wedge x-4 \leq 0$$

$$x \geq 2 \wedge x \geq 4; x \leq 2 \wedge x \leq 4.$$

$$x \geq 4; x \leq 2$$

Codomínio

$$y = \sqrt{x^2 - 6x + 8}$$

$$y^2 = x^2 - 6x + 8$$

$$x^2 - 6x + 8 - y^2 = 0$$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(8-y^2)}}{2}$$

$$x = \frac{6 \pm \sqrt{36 - 32 + 4y^2}}{2}$$

$$x = \frac{6 \pm \sqrt{4y^2 + 4}}{2}$$

$$4y^2 + 4 \geq 0 // (\frac{1}{4})$$

$$y^2 + 1 \geq 0$$

$$y^2 \geq -1$$

$$y \in \mathbb{R}$$

$$D_f: \forall x \in \mathbb{R} / x \geq 4 \wedge x \leq 2$$

$$C_f: \forall y \in \mathbb{R}$$

$$h) y = +\sqrt{x-2}$$

Domínio

$$y = \sqrt{x-2}$$

$$x-2 \geq 0$$

$$x \geq 2$$

Codomínio

$$y = \sqrt{x-2}$$

$$y^2 = x-2$$

$$x = y^2 + 2$$

$$D_f: \forall x \in \mathbb{R} / x \geq 2$$

$$C_f: \forall y \in \mathbb{R}$$

$$i) y = \log(x-6)$$

Domínio

$$y = \log(x-6)$$

$$x-6 > 0$$

$$x > 6$$

Codomínio

$$y = \log(x-6)$$

$$10^y = x-6$$

$$x = 10^y + 6$$

$$D_f: \forall x \in \mathbb{R} / x > 6$$

$$C_f: \forall y \in \mathbb{R}$$

$$j) y = \log(4-x^2)$$

Domínio

$$y = \log(4-x^2)$$

$$4-x^2 > 0 // (-1)$$

$$x^2 - 4 < 0$$

$$x^2 < 4$$

$$-2 < x < 2$$

Codomínio

$$y = \log(4-x^2)$$

$$10^y = 4-x^2$$

$$x^2 = 4-10^y$$

$$x = \sqrt{4-10^y}$$

$$4-10^y \geq 0$$

$$-10^y \geq -4 // (-1)$$

$$10^y \leq 4$$

$$y \leq \log(4)$$

$$D_f: \forall x \in \mathbb{R} / -2 < x < 2$$

$$C_f: \forall y \in \mathbb{R} / y \leq \log(4)$$

$$k) y = \log(x^2 + 1)$$

Domínio

$$y = \log(x^2 + 1)$$

$$x^2 + 1 > 0$$

$$x^2 > -1$$

$$x \in \mathbb{R}$$

Codominio

$$y = \log(x^2 + 1)$$

$$10^y = x^2 + 1$$

$$x^2 = 10^y - 1$$

$$x = \sqrt{10^y - 1}$$

$$10^y - 1 \geq 0$$

$$10^y \geq 1$$

$$y \geq \log(1)$$

$$y \geq 0$$

$$D_f: \forall x \in \mathbb{R}$$

$$C_f: \forall y \in \mathbb{R} / y \geq 0$$

$$l) y = \sqrt{x^2 - 4} + \log x$$

Domínio

$$y = \sqrt{x^2 - 4} + \log x$$

$$x^2 - 4 \geq 0$$

$$x^2 \geq 4$$

$$x \leq -2; x \geq 2$$

$$x > 0$$

Codominio

$$y = \sqrt{x^2 - 4} + \log x$$

Para $x \geq 2$ Ponto Mínimo

$$y_{(2)} = \sqrt{(2)^2 - 4} + \log(2)$$

$$y_{(2)} = 0 + \log(2)$$

$$y \geq \log(2)$$

$$D_f: \forall x \in \mathbb{R} / x \geq 2$$

$$C_f: \forall y \in \mathbb{R} / y \geq \log(2)$$

4) Hallar las funciones inversas de:

$$a) f(x) = 3x + 7$$

$$f(x) = 3x + 7$$

$$y = 3x + 7$$

$$x = \frac{y - 7}{3}$$

$$x - 7 = 3y$$

$$y = \frac{x - 7}{3}$$

$$f_{(x)}^{-1} = \frac{x - 7}{3}$$

$$b) f(x) = +\sqrt{x - 4}$$

$$f(x) = \sqrt{x - 4}$$

$$y = \sqrt{x - 4}$$

$$x = \sqrt{y - 4}$$

$$x^2 = y - 4$$

$$y = x^2 + 4$$

$$f_{(x)}^{-1} = x^2 + 4$$

$$c) f(x) = x^3 - 1$$

$$f(x) = x^3 - 1$$

$$y = x^3 - 1$$

$$x = \sqrt[3]{y + 1}$$

$$x + 1 = y^3$$

$$y = \sqrt[3]{x + 1}$$

$$f_{(x)}^{-1} = \sqrt[3]{x + 1}$$

$$d) f(x) = \frac{3x}{x - 5}$$

$$y = \frac{3x}{x - 5}$$

$$x = \frac{3y}{y - 5}$$

$$e) f(x) = x^2 - 2x + 5$$

$$y = x^2 - 2x + 5$$

$$x = y^2 - 2y + 5$$

$$x - 4 = y^2 - 2y + 5 - 4$$

$$\begin{aligned}
 x(y-5) &= 3y \\
 xy - 5x &= 3y \\
 xy - 3y &= 5x \\
 y(x-3) &= 5x \\
 y &= \frac{5x}{x-3}
 \end{aligned}$$

$$\begin{aligned}
 x-4 &= y^2 - 2y + 1 \\
 x-4 &= (y-1)^2 \\
 \sqrt{x-4} &= y-1 \\
 y &= \pm\sqrt{x-4} + 1
 \end{aligned}$$

$$\Leftrightarrow (a-b)^2 = a^2 - 2ab + b^2$$

$$f(x)^{-1} = \frac{5x}{x-3}$$

$$f(x)^{-1} = \pm\sqrt{x-4} + 1$$

5) Hallar: a) $(f+g)(x)$ b) $(f-g)(x)$ c) $(f \cdot g)(x)$ d) $(f/g)(x)$ e) $(f \circ g)(x)$

1) si: $f(x) = x^2 + 4$ ^ $g(x) = 3x^2 - 5x + 1$

a) $(f+g)(x) = f(x) + g(x)$

$$\begin{aligned}
 &= x^2 + 4 + 3x^2 - 5x + 1 \\
 &= 4x^2 - 5x + 5
 \end{aligned}$$

b) $(f-g)(x) = f(x) - g(x)$

$$\begin{aligned}
 &= x^2 + 4 - (3x^2 - 5x + 1) \\
 &= x^2 + 4 - 3x^2 + 5x - 1 \\
 &= -2x^2 + 5x + 3
 \end{aligned}$$

c) $(f \cdot g)(x) = f(x) \cdot g(x)$

$$\begin{aligned}
 &= (x^2 + 4) \cdot (3x^2 - 5x + 1) \\
 &= 3x^4 - 5x^3 + x^2 + 12x^2 - 20x + 4 \\
 &= 3x^4 - 5x^3 + 13x^2 - 20x + 4
 \end{aligned}$$

d) $(f/g)(x) = f(x) / g(x)$

$$= \frac{x^2 + 4}{3x^2 - 5x + 1}$$

e) $(f \circ g)(x) = (3x^2 - 5x + 1)^2 + 4$

$$\begin{aligned}
 &= 9x^4 + 25x^2 + 1 - 30x^3 + 6x^2 - 10x + 4 \\
 &= 9x^4 - 30x^3 + 31x^2 - 10x + 5
 \end{aligned}$$

2) si: $f(x) = 4x^3 + 2x - 1$ ^ $g(x) = 3x + 1$

a) $(f+g)(x) = f(x) + g(x)$

$$\begin{aligned}
 &= 4x^3 + 2x - 1 + 3x + 1 \\
 &= 4x^3 + 5x
 \end{aligned}$$

$$\begin{aligned}
 b) (f-g)(x) &= f(x) - g(x) \\
 &= 4x^3 + 2x - 1 - (3x + 1) \\
 &= 4x^3 + 2x - 1 - 3x - 1 \\
 &= 4x^3 - x - 2
 \end{aligned}$$

$$\begin{aligned}
 c) (f \cdot g)(x) &= f(x) \cdot g(x) \\
 &= (4x^3 + 2x - 1)(3x + 1) \\
 &= 12x^4 + 6x^2 - 3x + 4x^3 + 2x - 1 \\
 &= 12x^4 + 4x^3 + 6x^2 - x - 1
 \end{aligned}$$

$$\begin{aligned}
 d) (f/g)(x) &= f(x)/g(x) \\
 &= \frac{4x^3 + 2x - 1}{3x + 1}
 \end{aligned}$$

$$\begin{aligned}
 e) (f \circ g)(x) &= f(g(x)) = 4(3x + 1)^3 + 2(3x + 1) - 1 \\
 &= 4(27x^3 + 27x^2 + 9x + 1) + 2(3x + 1) - 1 \\
 &= 108x^3 + 108x^2 + 36x + 4 + 6x + 2 - 1 \\
 &= 108x^3 + 108x^2 + 42x + 5
 \end{aligned}$$

6) Hallar a) Dominio y Codominio b) Intersecciones en x y y c) Asíntotas Verticales y Horizontales.

$$1) y = \frac{1}{x-2}$$

a) Dominio

$$y = \frac{1}{x-2}$$

$$\begin{aligned}
 x-2 &\neq 0 \\
 x &\neq 2
 \end{aligned}$$

$$D_f: \forall x \in \mathbb{R} / x \neq 2$$

Codominio

$$y = \frac{1}{x-2}$$

$$\begin{aligned}
 y(x-2) &= 1 \\
 xy - 2y &= 1 \\
 xy &= 1 + 2y \\
 x &= \frac{1+2y}{y}
 \end{aligned}$$

$$y \neq 0$$

$$C_f: \forall y \in \mathbb{R} / y \neq 0$$

b) Para $x=0$

$$\begin{aligned}
 y &= \frac{1}{0-2} \\
 y &= -\frac{1}{2}
 \end{aligned}$$

$$(0, -\frac{1}{2})_{xy}$$

Para $y=0$

$$0 = \frac{1}{x-2}$$

$$0(x-2) = 1$$

$$0 = 1$$

$\emptyset$

c) Asíntota Vertical

$$x = 2$$

Asíntota Horizontal

$$y = 0$$

$$2) y = \frac{x-4}{x-2}$$

a) Dominio

$$y = \frac{x-4}{x-2}$$

$$x-2 \neq 0$$

$$x \neq 2$$

$$D_f: \forall x \in \mathbb{R} / x \neq 2$$

Codominio

$$y = \frac{x-4}{x-2}$$

$$y(x-2) = x-4$$

$$xy - 2y = x - 4$$

$$xy - x = 2y - 4$$

$$x(y-1) = 2y-4$$

$$x = \frac{2y-4}{y-1}$$

$$y-1 \neq 0$$

$$y \neq 1$$

$$C_f: \forall y \in \mathbb{R} / y \neq 1$$

b) Para $x=0$

$$y = \frac{0-4}{0-2}$$

$$y = \frac{-4}{-2} = \frac{-2}{-1}$$

$$y = 2$$

$$(0, 2) \text{ en } y$$

Para $y=0$

$$0 = \frac{x-4}{x-2}$$

$$0(x-2) = x-4$$

$$0 = x-4$$

$$x = 4$$

$$(4, 0) \text{ en } x$$

c) Asintota Vertical

$$x = 2$$

Asintota Horizontal

$$y = 1$$

$$3) y = \frac{x^2-9}{x^2-4}$$

a) Dominio

$$y = \frac{x^2-9}{x^2-4}$$

$$x^2-4 \neq 0$$

$$x^2 \neq 4$$

$$x = \pm\sqrt{4}$$

$$x \neq -2 \wedge x \neq 2$$

$$D_f: \forall x \in \mathbb{R} / x \neq -2 \wedge x \neq 2$$

Codominio

$$y = \frac{x^2-9}{x^2-4}$$

$$y(x^2-4) = x^2-9$$

$$x^2y - 4y = x^2 - 9$$

$$x^2y - x^2 = 4y - 9$$

$$x^2(y-1) = 4y-9$$

$$x^2 = \frac{4y-9}{y-1}$$

$$y-1 \neq 0$$

$$y \neq 1$$

$$C_f: \forall y \in \mathbb{R} / y \neq 1$$

b) Para $x=0$

$$y = \frac{0^2 - 9}{0^2 - 4}$$

$$y = \frac{-9}{-4}$$

$$y = \frac{9}{4}$$

$$(0, \frac{9}{4})_{3y}$$

Para $y=0$

$$0 = \frac{x^2 - 9}{x^2 - 4}$$

$$0(x^2 - 4) = x^2 - 9$$

$$0 = x^2 - 9$$

$$(x-3)(x+3) = 0$$

$$x-3=0; x+3=0$$

$$x=3; x=-3$$

$$(3,0)(-3,0)_{3x}$$

c) Asintota Vertical

$$x = -2 \wedge x = 2$$

Asintota Horizontal

$$y = 1$$

$$4) y = \frac{x}{x^2 - 4}$$

a) Dominio

$$y = \frac{x}{x^2 - 4}$$

$$x^2 - 4 \neq 0$$

$$x^2 \neq 4$$

$$x \neq \sqrt{4}$$

$$x \neq -2 \wedge x \neq 2$$

$$Df: \forall x \in \mathbb{R} / x \neq -2 \wedge x \neq 2$$

Codominio

$$y = \frac{x}{x^2 - 4}$$

$$y(x^2 - 4) = x$$

$$x^2 y - 4y = x$$

$$yx^2 - x - 4y = 0$$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4y(-4y)}}{2y}$$

Numerador

$$2 + 8y^2 \geq 0 \quad // (\frac{1}{2})$$

$$1 + 4y^2 \geq 0$$

$$4y^2 \geq -1$$

$$y^2 \geq -\frac{1}{4}$$

$$y \in \mathbb{R}$$

Denominador

$$2y \neq 0$$

$$y \neq 0$$

$$Cf: \forall y \in \mathbb{R} / y \neq 0$$

b) Para $x=0$

$$y = \frac{0}{0^2 - 4}$$

$$y = \frac{0}{-4}$$

$$y = 0$$

$$(0,0)_{3y}$$

Para $y=0$

$$0 = \frac{x}{x^2 - 4}$$

$$0(x^2 - 4) = x$$

$$x = 0$$

$$(0,0)_{3x}$$

c) Asintota Vertical

$$x = -2 \wedge x = 2$$

Asintota Horizontal

$$y = 0$$

$$5) y = \frac{x^2}{x-1}$$

a) Dominio

$$y = \frac{x^2}{x-1}$$

$$x-1 \neq 0$$

$$x \neq 1$$

$$D_f: \forall x \in \mathbb{R} / x \neq 1$$

Codominio

$$y = \frac{x^2}{x-1}$$

$$y(x-1) = x^2$$

$$yx - y = x^2$$

$$x^2 - xy + y = 0$$

$$x = \frac{-(-y) \pm \sqrt{(-y)^2 - 4y}}{2}$$

$$y^2 - 4y \geq 0$$

$$y(y-4) \geq 0 \Rightarrow y \geq 0 \wedge y-4 \geq 0; y \leq 0 \wedge y-4 \leq 0$$

$$y \geq 0 \wedge y \geq 4; y \leq 0 \wedge y \leq 4$$

$$y \geq 4; y \leq 0$$

$$C_f: \forall y \in \mathbb{R} / y \geq 4 \wedge y \leq 0$$

b) Para $x=0$

$$y = \frac{0^2}{0-1}$$

$$y = \frac{0}{-1}$$

$$y = 0$$

$$(0,0)_{I_y}$$

Para $y=0$

$$0 = \frac{x^2}{x-1}$$

$$0(x-1) = x^2$$

$$x^2 = 0$$

$$x = 0$$

$$(0,0)_{I_x}$$

c) Asintota Vertical

$$x = 1$$

Asintota Horizontal

No tiene

$$6) y^2 = \frac{x^2-4}{x^2-9}$$

a) Dominio

$$y = \sqrt{\frac{x^2-4}{x^2-9}}$$

$$x^2-4 \geq 0$$

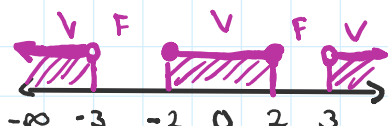
$$x^2 \geq 4$$

$$x = -2 \wedge x = 2$$

$$x^2-9 > 0$$

$$x^2 > 9$$

$$x = -3 \wedge x = 3$$



$$x > -3 \wedge -2 < 0 < 2 \wedge x > 3$$

$$D_f: \forall x \in \mathbb{R} / -3 < x \wedge -2 < 0 < 2; x > 3$$

Codominio

$$y^2 = \frac{x^2-4}{x^2-9}$$

$$y^2(x^2-9) = x^2-4$$

$$x^2y^2 - 9y^2 = x^2 - 4$$

$$x^2y^2 - x^2 = 9y^2 - 4$$

$$x^2(y^2-1) = 9y^2-4$$

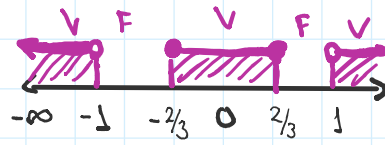
$$x^2 = \frac{9y^2-4}{y^2-1}$$

$$9y^2-4 \geq 0$$

$$y = \frac{2}{3} \wedge y = -\frac{2}{3}$$

$$y^2-1 > 0$$

$$y = 1 \wedge y = -1$$



$$y < -1 \wedge -\frac{2}{3} \leq y \leq \frac{2}{3} \wedge y > 1$$

$$C_f: \forall y \in \mathbb{R} / y < -1 \wedge -\frac{2}{3} \leq y \leq \frac{2}{3} \wedge y > 1$$

b)

Para $x=0$

$$y^2 = \frac{0^2 - 4}{0^2 - 9}$$

$$y = \sqrt{\frac{4}{9}}$$

$$y = \pm \frac{2}{3}$$

$$(0, -\frac{2}{3})(0, \frac{2}{3}) \cap y$$

Para $y=0$

$$0^2 = \frac{x^2 - 4}{x^2 - 9}$$

$$0(x^2 - 9) = x^2 - 4$$

$$0 = x^2 - 4$$

$$x^2 = 4$$

$$x = -2 \wedge x = 2$$

$$(-2, 0)(2, 0) \cap x$$

c)

Asintota Vertical

$$x = -3 \wedge x = 3$$

Asintota Horizontal

$$y = 1$$

NUEVA FUERZA
FINANCIERA